



The Methodology of Applying Inverse Distance Weighting Interpolation Method in Determining Normal Heights

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Abstract

This article explores the application of the Inverse Distance Weighting (IDW) interpolation method in determining normal heights, utilizing a network of points with measured ellipsoidal and normal heights by the use of the GNSS/leveling method. The IDW method, which assumes that points closer to each other are more alike, is employed to locate points within the correct quadrant and calculate their normal height based on the determined average height anomaly across the entire area. The accuracy of this method is validated using checkpoints. The article further discusses the methodology of normal height determination at a country level by using generated coefficients, representing the average difference between various height systems, which could be used as an integral to the interpolation methods used for generating a refined Digital Elevation Model (DEM) across the country. This study contributes to the ongoing discourse on the effective use of interpolation methods in geospatial analysis and DEM generation. It provides valuable insights into the practical application of the IDW method and its potential for enhancing the accuracy of normal height determination and DEM refinement. Importantly, such methods hold significant value for countries lacking gravimetric data, as they provide a viable means of determining normal heights in the absence of a local quasi-geoid model.

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Keywords

DEM; IDW; GNSS; Leveling; Normal Heights; Ellipsoidal heights

1. Introduction

Determining normal heights is of significant importance in various fields, including engineering and Earth system studies. Physical heights above sea level, which are most precisely determined through classical leveling, have crucial implications in these areas. However, classical leveling is time-consuming and expensive, necessitating the use of more efficient methods. In this context, the Global Navigation Satellite System (GNSS) has emerged as a powerful tool. GNSS leverages the benefits of all global navigation systems, providing more access and availability of signals to operators. Multiple GNSS constellations improve the availability of the navigation solution, saving time and money by delivering increased location accuracy at enhanced performance while determining highly accurate elevations using GNSS is still a challenging task facing engineers nowadays (Abdulrahman, 2021).

In the field of geospatial analysis, the accurate determination of normal heights and the generation of refined Digital Elevation Models (DEMs) are crucial for a wide range of applications. However, in areas where gravimetric data is lacking, finding reliable methods for height determination becomes a challenge. This is where the Inverse Distance Weighting (IDW) interpolation method comes into play. This article delves into the practical application of the IDW

method in determining normal heights, utilizing a network of points with measured ellipsoidal and normal heights obtained through the GNSS/leveling method (Mustafin et al, 2023).

The IDW interpolation method is a widely used technique in spatial analysis and geo-statistics. It is used to estimate values at unknown locations based on the values observed at known locations. In the case of height determinations, IDW is used to interpolate elevation values from a set of known height measurements. It assumes that the values closer to the unknown location have more influence on the estimated value than those farther away. The method assigns weights to each known value based on their proximity to the unknown location, with closer points having higher weights. IDW is particularly useful in height determinations because it takes into account the spatial relationships between data points. It can capture local variations in elevation and produce smooth surfaces that accurately represent the terrain. This is important for various applications, such as creating digital elevation models, flood modeling, and urban planning. The accuracy of IDW at a country level depends on several factors, including the density and distribution of the known height measurements. In areas with dense and evenly distributed data points, the accuracy can be high. However, in regions with sparse or unevenly distributed measurements, the accuracy may decrease. It is essential to consider the limitations of the method and complement it with other interpolation techniques or field surveys to improve accuracy at a country level (Habib et al, 2020).

2. Materials and Methods

Lebanon faces challenges in determining accurate normal heights due to the lack of gravimetric data and the absence of a local quasi-geoid model. The country's topography is characterized by a mixture of mountains and flat areas, which further complicates the task of accurately determining heights due to variations in gravitational pull. To address this issue, a study was conducted using the GNSS/leveling method in different areas with diverse topographic features. The chosen sites were located in Bekaa, Mount Lebanon, and North governorates and named Site B, site M, and Site N respectively. This method aimed to determine both ellipsoidal and normal heights, while also investigating the height anomaly in the areas (Al Shouny et al., 2023). Through this research, efforts are being made to improve the understanding and measurement of normal heights in Lebanon. The chosen areas for applying the GNSS/leveling method are shown in Figure 1.

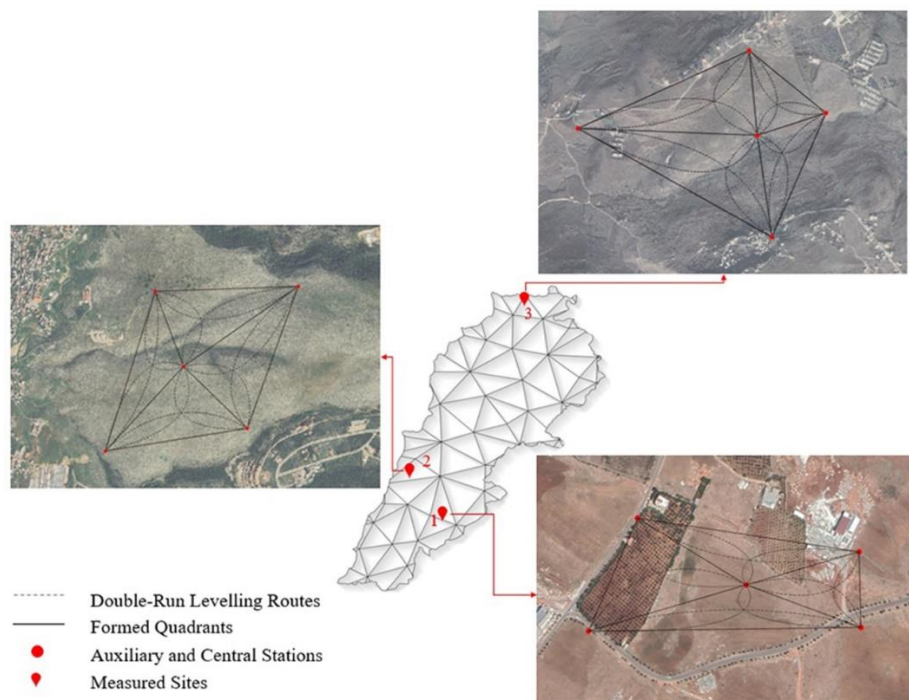


Figure 1. Location of the areas where the GNSS/ Levelling method is applied. Site B is located at Bekaa governorate, site M at Mount Lebanon governorate, and site N at North governorate

The methodology started with GNSS static measurements at the study areas where in each study area the located points were forming a quadrant with a central station. The project settings were carefully chosen and the GNSS

observations were carefully reviewed and cleaned to eliminate any unreliable data and satellites that were not functioning properly. The baselines were then processed and all of them met the required criteria for acceptance, with no baselines marked as problematic. An adjustment was conducted to identify any erroneous observations in the network and to provide error estimates for all observations (Liang et al, 2022). Upon completion of the GNSS measurements, the subsequent step entails determining the normal heights for each point through the use of a digital level. To guarantee precision, systematic errors such as instrumental and observational errors, atmospheric refraction, and Earth curvature were identified and rectified.

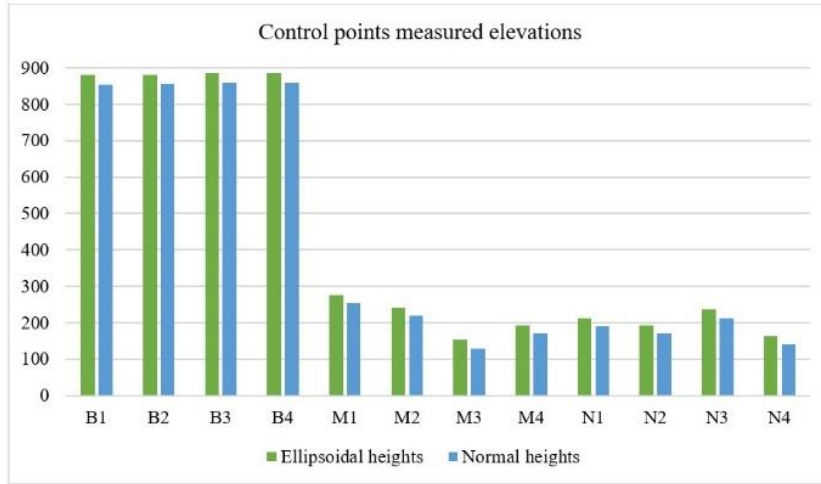


Figure 2. Measured ellipsoidal and normal heights for the control points in the study areas B, M, and N.

Once the normal heights were adequately adjusted, the values for height anomalies were computed for each site by subtracting the normal heights (H) from the ellipsoidal heights (h) at each point (Sjöberg, 2018). The findings revealed that elevation changes in flat regions exhibit relatively consistent and uniform patterns, while other areas displayed irregularities and fluctuations in the terrain, resulting in a non-linear correlation between the height anomaly values (Fig. 3).

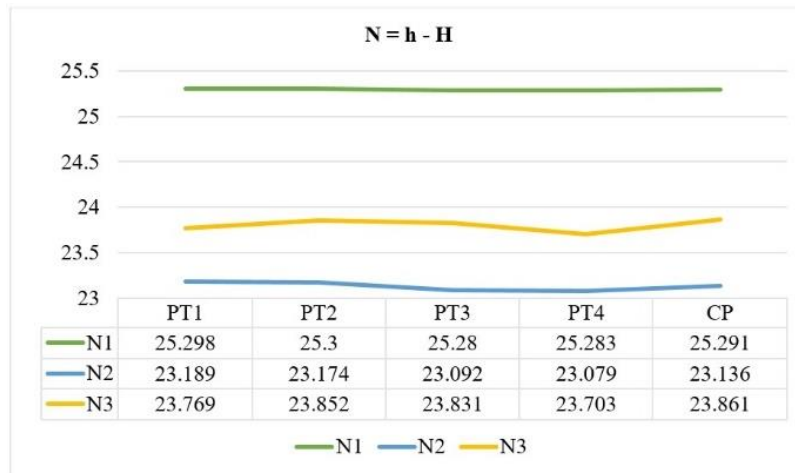


Figure 3. Variation in Height anomaly at each point in the study areas

To interpolate the data within each quadrant of the region of interest, the interpolation method employed was the inverse distance weighting (IDW) method. This mathematical technique is widely utilized for spatial interpolation purposes. The fundamental assumption of the IDW method is that objects or points near each other are more similar than those located further apart (Mulkal et al, 2019). To predict the value for any unmeasured location, the IDW method considers the measured values surrounding the prediction location. The measured values that are closest to the prediction location carry more weight and have a greater impact on the predicted value compared to those that are farther away (Setianto, 2013). The IDW method assumes that each measured point possesses a local influence that diminishes as the distance increases. As a result, it assigns higher weights to points that are near the prediction

location, while the weights progressively decrease as a function of distance (Hastaoğlu et al, 2022). Figure 4 provides a visual representation of the weights assigned to the data points. The weights window displays the list of weights assigned to each data point, which is utilized to generate a predicted value at the location indicated by the crosshair.

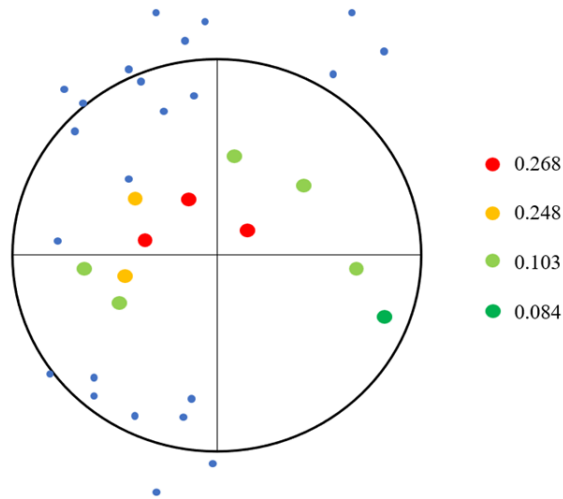


Figure 5. Illustration of Inverse Distance Weighting Interpolation method.

By adding each region of interest as a quadrant, the IDW method allowed for a more comprehensive interpolation analysis to be utilized for generating a DEM (Okolie et al, 2022). In this method, the central points were excluded from the interpolation process but used as checkpoints to assess the accuracy of the interpolation. Each checkpoint, assumed to have unknown normal heights, will be tested by the IDW interpolation to specify in which quadrant it is located, then, using normal heights of the quadrant vertices the height of each checkpoint will be determined.

3. Results

A DEM was generated for the three sites and shown in Figure 4.

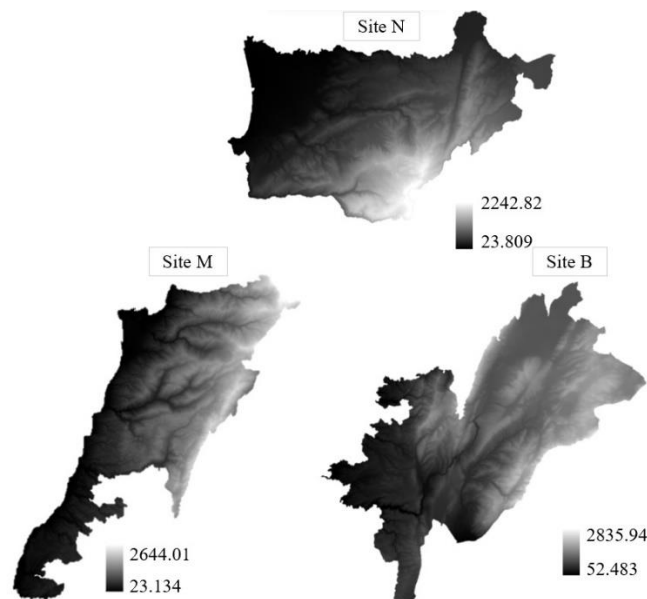


Figure 6. The created DEMs for the regions surrounding the study areas are based on the determined coefficients.

In Excel, the IDW formula was applied over the three quadrants. A checkpoint with known longitude, latitude, and ellipsoidal height is included. The objective is to calculate the normal height (H) of the checkpoint based on the known points. This formula helps determine the location of the checkpoint and in which quadrant it is situated. The "if" function is utilized in the formula, which allows the computation of the checkpoint's location. If the checkpoint falls within the correct quadrant, the formula directly calculates its normal height. However, if the checkpoint is located in any other quadrant, the formula will display "false" as the result. Three checkpoints were tested randomly,

and their initial values and the results are recorded in Table 2. The difference between actual normal heights and derived normal heights for the used checkpoints was in centimeters and sub-centimeters. The achieved results ensured the high accuracy of the adapted method in height determinations.

Table 1. Table 1. Comparison between ellipsoidal heights (h), normal heights (H), and heights derived from IDW for the control points.

Check point	Φ	Λ	h	H	H _{IDW}	ΔH
CP1	33.517137	35.670429	882.446	857.155	857.154	0.001
CP2	33.722852	35.466929	192.591	199.455	169.464	0.009
CP3	34.609521	36.131414	164.500	140.639	140.756	0.117

4. Conclusion

The GNSS/leveling geometric method was applied over three distinct areas in Lebanon having different topographic characteristics. Ellipsoidal and normal heights were measured, and height anomalies were determined. The differences in height anomalies reflect the topographic variations within the different sites. IDW interpolation method was chosen to be tested for normal height determinations, where checkpoints having known ellipsoidal and normal heights were used for testing the methodology. The difference between actual normal heights and derived normal heights for the checkpoints was in centimeters and sub-centimeters. This ensures the high accuracy of the adapted method. IDW could be further used at a country level if enough data exists. Thus, for Lebanon, with the current absence of gravimetric data, GNSS/leveling is the preferable method for determining normal heights. It could be applied over distinct areas to cover the overall territory in a well-distributed form. In this way, the IDW interpolation will function well with less distortions. Further steps might include the creation of a well-refined DEM, which will give access to normal height determination for any specified point within the territory having a known location and ellipsoidal height.

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Ethics approval

Not applicable.

Conflict of interest

The authors declare that there is no competing interest.

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